CS215 Assignment 5

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Question 1

As N increases the error in the Ml estimate Map1 estimate and Map2 estimate almost become equal, this is true because it is Gaussian
And we prefer MAP1 estimate because it gives the least error though the error has slightly higher Standard deviation.
The image is saved as $final.png$ in the results folder under Q1, Other photos are also attached.

Question 2

Here $y = (-1/\lambda)log(x)$ which gives $x = e^{-\lambda y}$.
Or $g^{-1}(y) = e^{-\lambda y}$ using this

$$P(Y = y) = P(X = e^{-\lambda y})|\frac{d(e^{-\lambda y})}{dy}| = \lambda e^{-\lambda y}$$

Likelihood function is

$$\prod_{i=1}^{N}(\lambda e^{-\lambda y_i}) = \lambda^N e^{-\lambda \sum_{i=1}^{N} y_i}$$

Differentiating it gives ML estimate as

$$\hat{\lambda}^{ML} = \frac{N}{\sum_{i=1}^{N} y_i}$$

$P(\lambda) = c\lambda^{\alpha-1}e^{-\beta\lambda}$
Now Likelihood function is where c is some constant.

$$c\lambda^{\alpha-1}e^{-\beta\lambda}\prod_{i=1}^{N}(\lambda e^{-\lambda y_i}) = c\lambda^{N+\alpha-1}e^{-\lambda(\beta+\sum_{i=1}^{N} y_i)}$$

Differentiating it gives estimate as

$$\hat{\lambda}^{PosteriorMean} = \frac{N+\alpha-1}{\beta+\sum_{i=1}^{N} y_i}$$

As N increases the error in the Ml estimate Map1 estimate and posterior mean estimate become equal which can be seen clearly from the formulas
And we prefer posterior mean estimate as its error is low and the deviation from the error is also low
The image is saved as $final.png$ in the results folder under Q2, Other photos are also attached

Question 3

By Bayesian Statistical Analysis,
posterior = $\frac{prior \times likelihood}{denominator}$
Draw N samples of the random variable X

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{\int_{\theta_m}^{\infty} P(X|\theta)P(\theta)d\theta}$$

Using P(X|θ) = $(\frac{1}{\theta})^N$ since N samples are drawn and P(θ) = $(\frac{\theta_m}{\theta})^\alpha$
We get

$$P(\theta|X) = \frac{n+\alpha-1}{\theta}(\frac{\theta_m}{\theta})^{(n+\alpha-1)} as \int_{\theta_m}^{\infty} \frac{\theta_m^\alpha}{\theta^{n+\alpha}}d\theta = \frac{n+\alpha-1}{\theta_m^{n-1}}$$

Now we can see that P(θ|X) is max at θ_m according to the above equation but θ must also be greater than the max value of the Random variable X
Therfore we get

$$\hat{\theta}^{ML} = max(X_i, i = [n]) \ and \ \hat{\theta}^{MAP} = max(\theta_m, \hat{\theta}^{ML})$$

If the true value of θ is less than the value of θ_m, then we can say that $\hat{\theta}^{MAP} = \theta_m$ in which case the estimate is not as good because we are getting the prior parameter value assumed which is not good , otherwise $\hat{\theta}^{MAP} = \hat{\theta}^{ML}$ is observed since X can take values more than θ_m which means that maximum-a-posterior estimate turns out to be same as ML estimate , The first case is not desirable where as the second is much desirable.
Now coming to estimating the mean using the posterior pdf determined above ,

$$E(\theta) = \int_{\theta_m}^{\infty} \theta P(\theta|X)d\theta$$

$$E(\theta) = \int_{\theta_m}^{\infty} \theta \frac{n+\alpha-1}{\theta}\frac{\theta_m}{\theta}^{(n+\alpha-1)} d\theta$$

$$E(\theta) = \frac{n+\alpha-1}{n+\alpha-2}\theta_m$$

Now as n approaches infinity the value of $E(\theta) = \theta_m$ doesn't approach ML estimate which is not desirable and is bad estimate , it is as if we are doing the analysis for waste , because the choice of the parameters is totally upto us and we can use a very bad value to get the same value as a output. So this case is not desirable.